EX NAVODAYAN FOUNDATION
(Registered Under Indian Trust Act 1882) Reg. No. : 2016, 43
B/36/43 46M Brahmanand Colony, Durgakund, Varanasi (UP) 221005
Mob.: 6391500102 Email Id : exnavodayanfoundation@gmail.com

Full Syllabus

JEE-Main

Paper-2

Test Date: **M.M:300**

TEST INSTRUCTIONS

1. The test is of **3 hours** duration.
2. The test booklet consists of **75 questions**.
3. The maximum marks are **300**.
4. All questions are compulsory.
5. There are three parts in the questions paper consisting of Physics, Chemistry and Mathematics having **25 questions in each part**.

Each Parts Contains –

- 20 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct. All questions are carrying **+4 marks** for right answer and **-1 mark** for wrong answer.
- 05 questions with answer as **numerical value** all questions are carrying **+4 marks** for right answer and **–1 marks** for wrong answers.

Name of the Candidate (in Capital Letter): _____

Registration No. _____

Invigilator Signature

_____

Physics

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. A triangular lamina of area 'A' and height h is immersed in a liquid of density ρ in a vertical plane with its base on the surface of the liquid. The thrust on the lamina is

   (a) $\frac{1}{2}A\rho gh$ (b) $\frac{1}{3}A\rho gh$ (c) $\frac{1}{6}A\rho gh$ (d) $\frac{2}{3}A\rho gh$

2. An electric kettle taken 4A current at 220 V. Then time taken to boil 1 kg of water from temperature 20°C will be _____ min. If the temperature of boiling water is 100°C.

   (a) 6.35 min (b) 4.35 min (c) 8.35 min (d) 10.35 min

3. When a spring is stretched by a distance x, it exerts a force, give by $F = (-5x - 16x^3)N$. The work done (in joule), when the spring is stretched from 0.1 m to 0.2 m is

   (a) 0.087 J (b) 8.7 J (c) 87 J (d) 0.87 J

4. A closed gas cylinder is divided into two parts by a piston held tight. The pressure and volume of gas in two parts respectively are (P, 5V) and (10P, V). If now the piston is left free and the system undergoes isothermal process, then the volume of the gas in two parts respectively are:

   (a) 2V, 4V (b) 3V, 3V (c) 5V, V (d) 4V, 2V

5. A stone weighing 1 kg and sliding on ice with a velocity of 2 m/s is stopped by friction in 10 sec. The force of friction (assuming it to be constant) will be

   (a) –20 N (b) –0.2 N (c) 0.2 N (d) 20 N

6. If a circular coil with a radius of 5 cm, 250 turns, and a resistance of 8Ω has a magnetic field directed perpendicular to its plane that is decreasing at a rate of 0.6 T/S then the induced current in the coil is

   (a) 0.304 A (b) 0.561 A (c) 0.147 A (d) 0.23 A

7. In H-like atom electron makes transition from an energy level with quantum number n to another with quantum number n – 1. If n >> 1, the frequency of radiation emitted is proportional to

   (a) $\frac{1}{n^2}$ (b) $\frac{1}{n^{3/2}}$ (c) $\frac{1}{n^3}$ (d) $\frac{1}{n}$

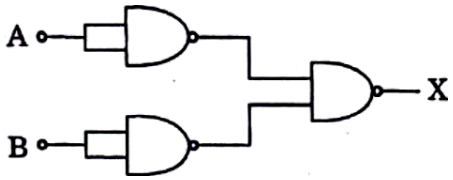
8. The ratio between the kinetic energy to the total energy of on electron in Bohr orbit is

   (a) 1 : –1 (b) –1 : 1 (c) 1 : 2 (d) 2 : –1

9. The wavelength of light emitted in the visible region by the H_e^+ ions after collision with H-atoms is

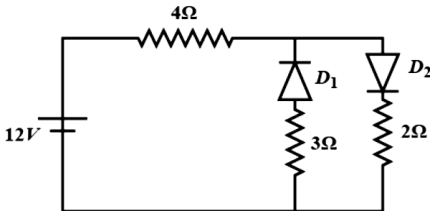
   (a) 6.5×10^{-7} m (b) 5.6×10^{-7} m (c) 4.8×10^{-7} m (d) 4.0×10^{-7} m

10. The combination of gases shown below yield



(a) NAND gate (b) OR gate (c) NOT gate (d) XOR gate

11. The circuit has two oppositely connected ideal diodes is parallel. What is the current following in the circuit.



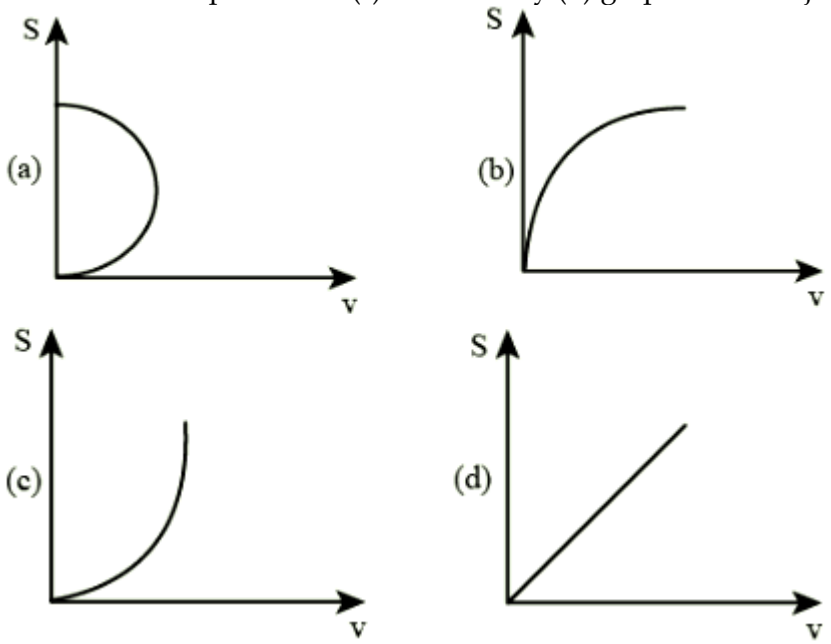
(a) 1.33 A (b) 1.71 A (c) 2 A (d) 2.31 A

12. In case of P-N junction diode at high value of reverse bias, the current rises sharply. The value of reverse bias is known as
(a) Cut of voltage (b) Zener voltage (c) Inverse voltage (d) Critical voltage

13. A uniformly charged conducting sphere of 2.4 m diameter, has a surface charged density of 80 $\mu C/m^2$. What is the total electron flask learning the surface of the sphere.
(a) 1.6×10^8 Nm^2/C (b) 3.2×10^8 Nm^2/C (c) 4.8×10^8 Nm^2/C (d) 6.4×10^8 Nm^2/C

14. If the binding energy per nucleon in ${}_3Li^7$ & ${}_2He^4$ nuclei one 5.60 MeV & 7.06 MeV respectively then in the reaction Ep + ${}_3Li^7 \rightarrow 2\,{}_2He^4$ energy of photon (Ep) must be _____
(a) 39.2 MeV (b) 28.24 MeV (c) 17.28 MeV (d) 1.46 MeV

15. An object is moving with uniform acceleration which is parallel to its instantaneous direction of motion. The displacement (s) and velocity (v) graph of this object is



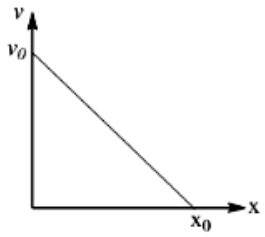
16. From a balloon rising vertically upward 5 ms^{-1} a stone is thrown up at 10 m/s^{-1} relative to balloon it's velocity with respect to ground after 2 sec. is (assume g = 10 m/s^2)

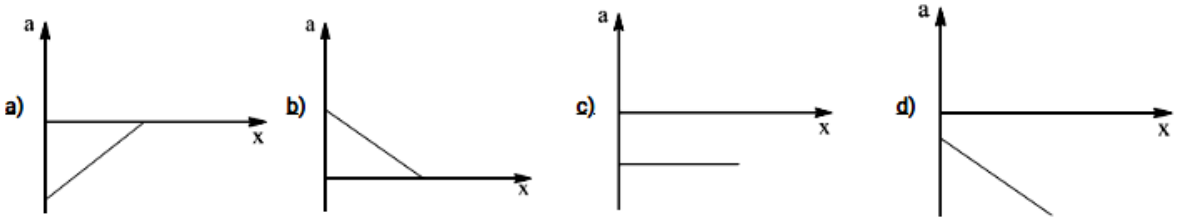
(a) 0 m/s (b) 20 m/s (c) 10 m/s (d) 5 m/s

17. Two bodies of mass 3 kg and 4 kg are suspended at the ends of massless string passing over a frictionless pulley. The acceleration of the system is ($g = 9.8\ m/s^2$)

(a) $4.9\ m/s^2$ (b) $2.45\ m/s^2$ (c) $1.4\ m/s^2$ (d) $9.5\ m/s^2$

18. The given graph shows the variation of velocity with displacement. Which one of the graph below correctly represents the variation of acceleration with displacement?





19. The electric field in a region is given by $\vec{E} = a\hat{i} + b\hat{j}$. Here 'a' and 'b' are constants. Find the net flux passing through square area of side 'ℓ' parallel to y – z plane is

(a) $a\ell^2$ (b) $\frac{a\ell^2}{2}$ (c) $2a\ell^2$ (d) $3a\ell^2$

20. When current in a coil changes from 5A to 2A in 0.1 sec., an average voltage of 50V is produced. The self inductance of the coil is

(a) 0.67 H (b) 1.67 H (c) 3 H (d) 6 H

(Integer Type Questions)

This Section contains **05 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. A solid sphere of mass 1 kg, radius 10 cm rolls down an inclined plane of height 7 m. The velocity of its center as it reaches the ground level is _____ (in m/s)

22. In a transformer, the number of turns in primary are 140 and that in secondary are 280. If the current in primary winding is 4A, then what is the secondary winding is _____ (in ampere)

23. The heal is flowing through a rod of length 50 cm and area of cross-section 5 cm^2. Its ends are respectively at 25°C and 125°C. The coeff of thermal conductance of material of rod is 0.092 kcal/ms °C. The temperature gradient of the rod is _____ (°C/cm)

24. Length of string tied to two rigid supports is 40cm Maximum length (wavelength in cm) of a stationary wave produced on it is _____ (in cm)

25. The phase difference between the instantaneous velocity and acceleration of a particle executing simple harmonic motion is $n \times 10\pi$. Then n = _____

Chemistry

(Single Correct Choice Type)

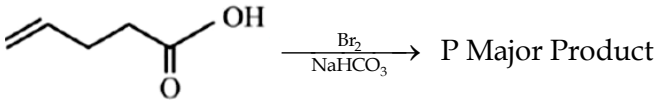
This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

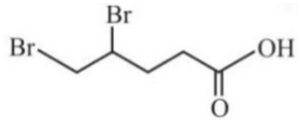
1. The pair from the following pairs having both compounds with net non-zero dipole moment is

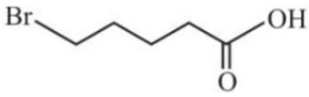
   (a) 1,4-dichlorobenzene, 1,3-dichlorobenzene (b) CH_2Cl_2, $CHCl_3$

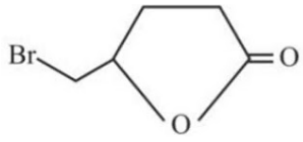
   (c) Benzene, anisidine (d) Cis-butene, trans-butene

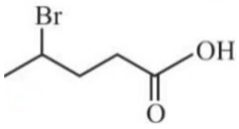
2. Major product 'p' formed in the following reaction is



(a) 

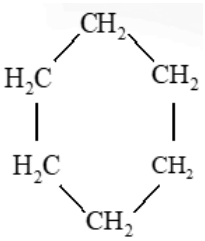
(b) 

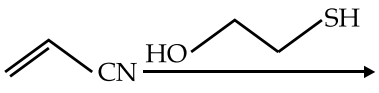
(c) 

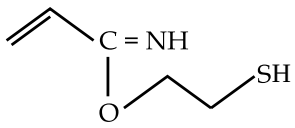
(d) 

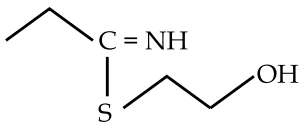
3. In the following reaction 'x' is $CH_3(CH_2)_4CH_3 \xrightarrow[\Delta]{\text{Anhy. } AlCl_3,\ HCl} \underset{\text{Major Product}}{'X'}$

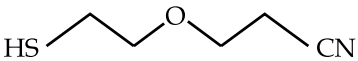
   (a) $CH_3(CH_2)_4CH_2Cl$ (b) $CH_3-\underset{\underset{CH_3}{|}}{CH}-(CH_2)_2CH_3$

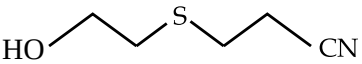
   (c) $ClCH_2-(CH_2)_4-CH_2Cl$ (d) 

4. The major product for the following reaction is 

(a) 

(b) 

(c) 

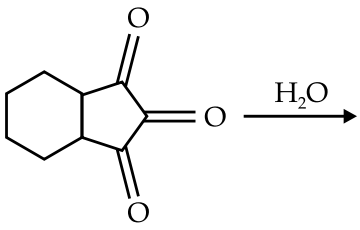
(d) 

5. $H_3C-CH_2-\underset{\underset{OH}{|}}{CH}-CH_3 \xrightarrow[(iii)\ D_2O]{(i)NaI,H_3PO_4,\ (ii)\ Mg,\ Dry\ ether} \underset{\text{Product}}{[X]}$

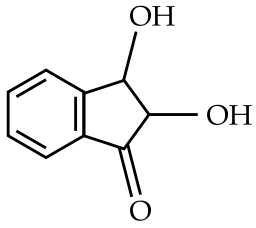
   Product [X] formed in the above reaction is

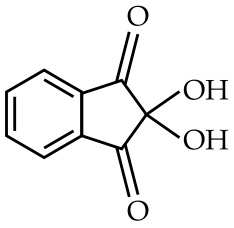
   (a) $H_3C-CH=CH-CH_3$ (b) $H_3C-CH_2-CH=CH_2$

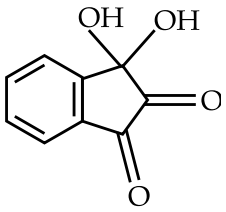
   (c) $H_3C-CH_2-\underset{\underset{D}{|}}{CH}-CH_3$ (d) $H_3C-CH_2-\underset{\underset{OH}{|}}{\overset{\overset{H}{|}}{C}}-CH_3$

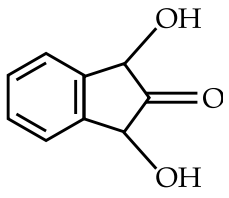
6.  ‘A’ major product

‘A’ formed in the above reaction is

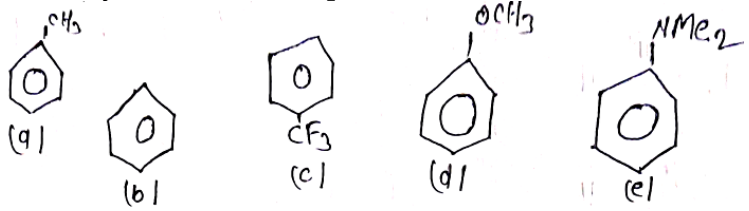
(a) 

(b) 

(c) 

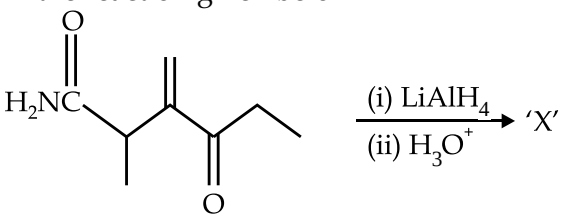
(d) 

7. Decreasing order of reactivity towards electrophilic substitution for the following compounds is

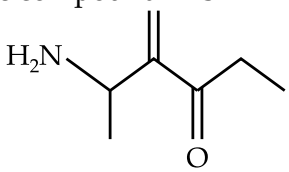


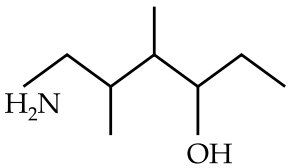
(a) a > d > e > b > c (b) e > d > a > b > c (c) c > b > a > d > e (d) d > a > e > c > b

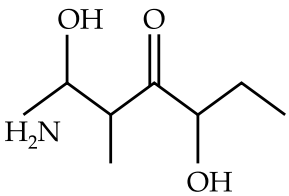
8. In the reaction given below

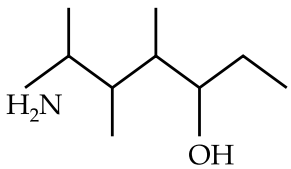


The compound X is

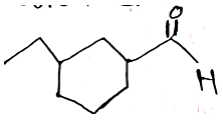
(a) 

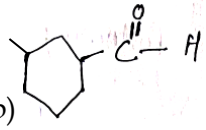
(b) 

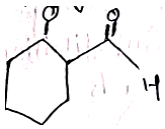
(c) 

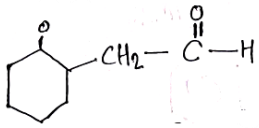
(d) 

9. Correct structure of γ-methylcyclohexane carbaldehyde is

(a) 

(b) 

(c) 

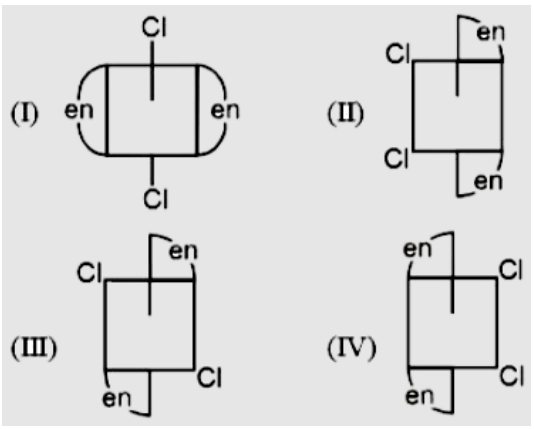
(d) 

10. $[CO(NH_3)_5NO_2]Cl_2$ and $[CO(NH_3)_5(ONO)]Cl_2$ are related to each other as

(a) Geometrical isomers (b) Optical isomers

(c) Linkage isomers (d) Coordination isomers

11. Identify the geometrical isomers of the following



(a) I with III (b) II with IV (c) I with II (d) None of these

12. Which of the following compounds show optical isomerism?

1. Cis-$[Co(NH_3)_4Cl_2]^+$ 2. Trans-$[Co(en)_2Cl_2]^+$ 3. Cis-$[Co(en)_2Cl_2]^+$ 4. $[Co(en)_3]^{3+}$

Select the correct answer using of codes given below:

(a) 1 and 2 (b) 2 and 3 (c) 3 and 4 (d) 1, 3 and 4

13. A protein 'X' with molecular weight of 70,000 u, on hydrolysis gives amino acids. One of these amino acid is

(a) $NH_2-CH_2-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_2CH_2COOH$

(b) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_2-\underset{\displaystyle NH_2}{\underset{|}{CH}}-COOH$

(c) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-\underset{\displaystyle NH_2}{\underset{|}{CH}}-CH_2COOH$

(d) $CH_3-\underset{\displaystyle NH_2}{\underset{|}{\underset{\displaystyle CH_3}{\underset{|}{C}}}}-CH_2-CH_2COOH$

14. Number of cyclic tripeptides formed with 2 amino acid A and B is

(a) 3 (b) 4 (c) 2 (d) 5

15. Consider the following first order competing reactions:

$X \xrightarrow{k_1} A + B$ and $Y \xrightarrow{k_2} C + D$

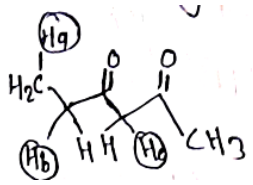
If 50% of the reaction of X was completed when 96% of the reaction of Y was completed, the ratio of their rate constants (K_2/K_1) is

(a) 4.06 (b) 0.215 (c) 1.1 (d) 4.65

16. The decomposition of N_2O_5 in CCl_4 was followed by measuring the volume of O_2 gas evolved: $2N_2O_5\ (CCl_4) \rightarrow 2N_2O_4\ (CCl_4) + O_2(g)$. The maximum volume of O_2 gas obtained was 100 cm^3. In 500 minutes, 90 cm^3 of O_2 were evolved. The first order rate constant (in min^{-1}) for the disappearance of N_2O_5 is:

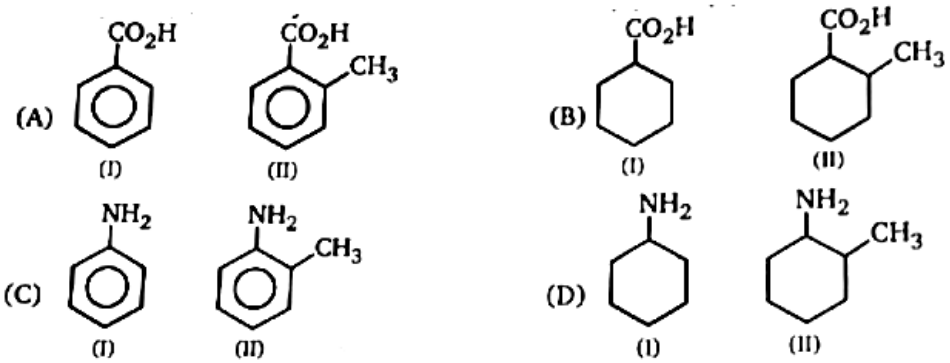
(a) $\frac{2.303}{500}$ (b) $\frac{2.303}{500}\log\frac{100}{90}$ (c) $\frac{2.303}{500}\log\frac{90}{100}$ (d) $\frac{100}{10\times500}$

17. Rank the hydrogen atoms (H_a, H_b, H_c) present in the following molecule in decreasing order of their acidic strength,



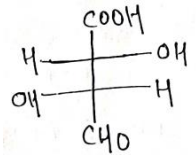
(a) a > b > c (b) b > a > c (c) b > c > a (d) c > b > a

18. In the given pair identify most acidic compound in (A & B) most basic in (C) and (D).



(a) A-I, B-II, C-I, D-II (b) A-II, B-I, C-I, D-II (c) A-II, B-II, C-II, D-II (d) A-I, B-II, C-I, D-I

19. The configurations of the carbon atoms C_2 and C_3 in the following compound are respectively.



(a) R, R (b) S, S (c) R, S (d) S, R

20. The compound that is chiral is

(a) 3-methyl-3-hexene (b) 4-chloro-1-methycyclohexane

(c) 2-phenylpentane (d) 1,3-disopropylbenzene

(Integer Type Questions)

This Section contains **05 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. The rate of decomposition of $NH_3(g)$ at 10 atm on platinum surface is zero order. What is the formation (in M min^{-1}) of $H_2(g)$. If rate constant of reaction $2NH_3(g) \rightarrow N_2(g) + 3H_2(g)$ is 2.0 M min^{-1}?

22. How many faradays are required for reduction of 1 mol $C_6H_5NO_2$ into $C_6H_5NH_2$?

23. The vapour pressure of two pure liquids A and B are 5 and 10 torr respectively. Calculate the total pressure of the solution (in torr) obtained by mixing 2 mole of A and 3 mole of B.

24. In the ground state of atomic Fe (z = 26) the spin only magnetic moment is _____ $\times 10^{-1}$ BM. (Round off to the nearest integer) [Given : $\sqrt{3} = 1.73, \sqrt{2} = 1.41$]

25. Total number of acidic oxides among N_2O_3, NO_2, N_2O, Cl_2O_7, SO_2, CO, CaO, Na_2O and NO is __.

Mathematics

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. Let $f : R \to R$ be a function given by

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x^2} & x < 0 \\ \alpha & x = 0, \text{ where } \alpha, \beta \in R. \\ \dfrac{\beta\sqrt{1-\cos 2x}}{x} & x > 0 \end{cases}$$

If f is continues at x = 0, then $4\alpha^2 + 8\beta^2$ is equal to

(a) 160 (b) $\frac{81}{2}$ (c) 162 (d) $\frac{81}{4}$

2. Distance of the point (2, 5) from the line 3x + y + 4 = 0 measured parallel to the line 3x – 4y + 8 = 0 is

(a) 15/2 (b) 9/2 (c) 5 (d) None

3. The solution of differential equation $(x^2-1)\dfrac{dy}{dx} + 2xy = \dfrac{1}{x^2-1}$ is

(a) $y(x^2-1) = \dfrac{1}{2}\log\left|\dfrac{x-1}{x+1}\right| + C$ (b) $y(x^2+1) = \dfrac{1}{2}\log\left|\dfrac{x-1}{x+1}\right| - C$

(c) $y(x^2-1) = \dfrac{5}{2}\log\left|\dfrac{x-1}{x+1}\right| + C$ (d) None of these

4. Determine the values of x satisfying the equality
$|(x^2 + 4x + 9) + (2x – 3)| = |x^2 + 4x + 9| + |2x – 3|$

(a) $x \le \frac{3}{2}$ (b) $x \ge \frac{3}{2}$ (c) $x = \frac{3}{2}$ (d) None

5. Consider two curves $C_1 : y = 1 + \cos x$ and $C_2 : y = 1 + \cos(x – \alpha)$ for $\alpha \in \left(0, \dfrac{\pi}{2}\right)$, $x \in [0, \pi]$. Find the value of α, for which the area of the figure bounded by the curves C_1, C_2 & x = 0 is same as that of the figure bounded by C_2, y = 1 and $x = \pi$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) none

6. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \le i, j \le 3$. If the determinant of p is 2, then the determinant of the matrix Q is

(a) 2^{10} (b) 2^{11} (c) 2^{12} (d) 2^{13}

7. $\displaystyle\int \frac{\cos x + \sqrt{3}}{1 + 4\sin\left(x + \frac{\pi}{3}\right) + 4\sin^2\left(x + \frac{\pi}{3}\right)} dx$ is

(a) $\dfrac{\cos x}{1 + 2\sin\left(x + \frac{\pi}{3}\right)} + C$ (b) $\dfrac{\sec x}{1 + 2\sin\left(x + \frac{\pi}{3}\right)} + C$

(c) $\dfrac{\sin x}{1 + 2\sin\left(x + \frac{\pi}{3}\right)} + C$ (d) $\dfrac{1}{2}\tan^{-1}\left(1 + 2\sin\left(x + \frac{\pi}{3}\right)\right) + C$

8. The expression $P(x)=\left(\sqrt{x^5-1}+x\right)^7-\left(\sqrt{x^5-1}-x\right)^7$ is polynomial of degree.

(a) 16 (b) 18 (c) 20 (d) 27

9. The number of ways in which three distinct numbers are in A.P. can be selected from the set {1, 2, 3, …., 24} is equal to

(a) 66 (b) 132 (c) 198 (d) None of these

10. A and B play a game of tennis. The situation of the game is as follows. If one scores two consecutive points after a deuce he wins. If loss of a point is followed by win of a point, it is deuce. The chance of a server to win a point is $\frac{2}{3}$. The game is at deuce and A is serving. Probability that A will win the match is (serves are changed after each point scored)

(a) $\frac{3}{5}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{4}{5}$

11. Let $\vec{\alpha}=a\hat{i}+b\hat{j}+c\hat{k}$ and $\vec{\beta}=b\hat{i}+c\hat{j}+a\hat{k}$, where a, b, c ∈ R. If θ be the angle between $\vec{\alpha}$ and $\vec{\beta}$ then

(a) $\theta\in\left[0,\frac{\pi}{2}\right]$ (b) $\theta\in\left[0,\frac{2\pi}{3}\right]$ (c) $\theta\in\left[\frac{2\pi}{3},\pi\right]$ (d) None of these

12. The value of $\sum_{\alpha=1}^{1024}\left[\log_2^{\alpha}\right]$ is equal to ([.] denotes the greatest integer function)

(a) 8192 (b) 8204 (c) 8194 (d) None of these

13. The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14 then the product of the remaining two observations is

(a) 40 (b) 45 (c) 49 (d) 48

14. If $\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=a^2-b^2$ and $\frac{ax\sin\theta}{\cos^2\theta}-\frac{by\cos\theta}{\sin^2\theta}=0$ then $(ax)^{2/3}+(by)^{2/3}=$

(a) a^2-b^2 (b) $(a^2-b^2)^{2/3}$ (c) a^2+b^2 (d) None of these

15. If the set $\left\{\text{Re}\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right):z\in C,\text{Re}(z)=3\right\}$ is equal to the interval [α, β], then 24 (β – α) is equal to

(a) 36 (b) 27 (c) 30 (d) 42

16. If the mirror image of the point P(3, 4, 9) in the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-2}{1}$ is (α, β, γ) then 14(α + β + γ) is

(a) 102 (b) 138 (c) 108 (d) 132

17. Let C : $x^2+y^2=4$ and C^1 : $x^2+y^2-4x+9=0$ be two circles. If the set of all values of d so that the circles C and C^1 intersect at two distinct points, is R – [a, b], then the point (8a + 12, 16b – 20) lies on the curve

(a) $x^2+2y^2-5x+6y=3$ (b) $5x^2-y=-11$
(c) $x^2-4y^2=7$ (d) $6x^2+y^2=42$

18. Find a if domain of function $f(x)=\sqrt{x^2+ax+4}$ is all real

(a) (–2, 2) (b) [–2, 4] (c) [–4, 4] (d) (–4, 4)

19. If [x] represents the greatest integer less than or equal to x. If all the value of x such that the product $\left[x-\frac{1}{2}\right]\left[x+\frac{1}{2}\right]$ is prime, belongs to the set $[x_1, x_2) \cup [x_3, x_4)$, find the value of $x_1^2+x_2^2+x_3^2+x_4^2$

(a) 22 (b) 44 (c) 11 (d) None of these

20. Let $A=\begin{bmatrix} 4\sec^2\theta & 1 & 0 \\ 0 & 3\tan^2\theta & 1 \\ 0 & 1 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} \cot^2\theta & 2 & 0 \\ 1 & 3\operatorname{cosec}^2\theta & 1 \\ 1 & 1 & 2 \end{bmatrix}$ then minimum value of tr(AB) is (where tr(A) demotes trace of square matrix A)

(a) 12 (b) 20 (c) 32 (d) 64

(Integer Type Questions)

This Section contains **10 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. The number of points of discontinuity of f(x) in [0, 2] where $f(x)=\begin{cases} [\cos \pi x] & x \le 1 \\ |2x-3|[x-2] & x>1 \end{cases}$ ([.] denotes the greatest integer function), are

22. The sum of coefficients of even powers of x in the expansion of $\left(x+\frac{1}{x}\right)^{11}$ is

23. If $f(x)=\int_0^x \frac{dt}{(f(t))^2}$ and $\int_0^2 \frac{dt}{(f(t))^2}=(6)^{1/3}$ then f(9) is equal to

24. From a point P(λ, λ, λ) perpendicular PQ and PR are drawn respectively on the lines $\frac{x+7}{-6}=\frac{y-6}{7}=z$ and $\frac{7-x}{2}=y-2=z-6$. If P is such that ∠QPR is 90°, then the sum of possible values of λ is k, then value of 127 k is

25. Point O is the center of the ellipse with major axis AB and minor axis CD. Point F is one of the focus of this ellipse. If OF = 6, and the diameter of inscribed circle of ΔOCF is 2, then the value of $\frac{AB.CD}{13}$ is

Answer – key -

Physics

1. b
2. a
3. a
4. a
5. b
6. a
7. c
8. a
9. c
10. b
11. c
12. b
13. a
14. c
15. c
16. d
17. c
18. a
19. a
20. b
21. 10
22. 2
23. 2
24. 80
25. 5

Chemistry

1. b
2. c
3. b
4. d
5. c
6. b
7. b
8. b
9. B
10. C
11. C
12. C
13. B
14. B
15. D
16. A
17. D
18. B
19. A
20. C
21. 6
22. 6
23. 8
24. 49
25. 4

Math

1. c
2. c
3. a
4. b
5. a
6. d
7. c
8. a
9. b
10. c
11. b
12. b
13. d
14. b
15. c
16. c
17. d
18. c
19. c
20. c
21. 3
22. 0
23. 3
24. 520
25. 945

Physics

1.

Sol Thrust on dome = Pressure at Centroid × Area

$= \frac{h\rho g}{2} \times A = \frac{1}{3} A\rho gh.$

2.

Sol $P \times t = mc\Delta\theta \Rightarrow t = \frac{4200 \times m \times \Delta\theta}{VI} = 6.35 \text{ min}$

3.

Sol $F = -5x - 16x^3 = -(5 + 16x^2)x = -kx$

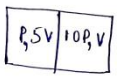
$\Rightarrow k = 5 + 16x^2$

Work done, $W = \frac{1}{2} k_2 x_2^2 - \frac{1}{2} k_1 x_1^2$

$= 0.087 J$

4.

Sol



When piston is allowed to move the gases are kept separate but the pressure has to be equal.

$\because P_1 = P_2 \quad PV = n_1 RT$

$\frac{n_1 RT}{x} = \frac{n_2 RT}{6V - x}$

$\Rightarrow x = 2V \text{ & } (6V - 2V) = 4V$

5.

Sol $v = u + at \Rightarrow a = -0.2 m/s^2$

Frictional force, $f = 1 \times (0.2) = -0.2 N$

6.

Sol $|\mathcal{E}| = N\frac{d\Phi_B}{dt} = N\pi R^2 \cdot \frac{dB}{dt} = 1.18 V$

$N = 250, R = 50 cm, \quad i = \frac{\mathcal{E}}{R} = \frac{1.18 V}{8} = 0.147 A$

7.

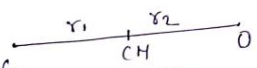
Sol $f \propto \text{can be written as,}$

$V \propto \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$

$\propto \frac{2(n-1)}{n^2(n-1)^2} \Rightarrow V \propto \frac{1}{n^3}$

8.

Sol $I = m_1 r_1^2 + m_2 r_2^2$



$I = m_1\left(\frac{m_2 d}{m_1 + m_2}\right)^2 + m_2\left(\frac{m_1 d}{m_1 + m_2}\right)^2 = \frac{m_1 m_2 d^2}{m_1 + m_2}$

$d = \left[\frac{I(m_1 + m_2)}{m_1 m_2}\right]$

$= 1.28 \times 10^{-10} m.$

9.

Sol $\Delta E = -4\left(13.6 - \frac{13.6}{16}\right) eV = -12.75 eV$

$\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = 4.905 \times 10^{-7} m$

10.

Sol

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

$\Rightarrow$ OR gate

11.

Sol $i = \frac{V}{R} = \frac{12}{4 + 2} = 2A$

12.

Solution:

In a p-n junction diode at high value of reverse bias, the current rises sharply due to zener breakdown. This high value of reverse bias is called zener voltage.

13.

Solution

Given, diameter of sphere =2.4m

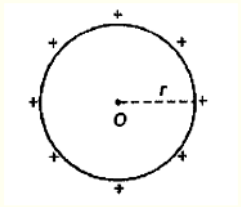
Radius of sphere r = $\frac{2.4}{2}$ = 1.2m

Surface charge density = $\frac{\text{Chrge}}{\text{Surface area}}$

$\sigma = \frac{q}{4\pi r^2}$

$q = \sigma \times 4\pi r^2 = 80 \times 10^{-6} \times 4 \times 3.14 \times 1.2 \times 1.2$

$q = 1.4 \times 10^{-3} C$



Total flux = ϕ

Using Gauss's theorem,

Total flux leaving the surface $\phi = \frac{\text{Total charge}}{\varepsilon_0}$

$\varepsilon_0 = \frac{q}{\varepsilon_0} = \frac{1.45\times10^{-3}}{8.854\times10^{-12}}$

$\phi = 1.6 \times 10^8 N - m^2/C$

Thus, the flux leaving the surface of sphere is $1.6 \times 10^8 N - m^2/C$.

14.

Sol: KE of photon + B.E of $_3Li^7$ = 2 × BE of $_2He^4$

$E_p = 8 \times 7.06\ MeV - 7 \times 5.06\ MeV$

$= 17.28\ MeV.$

15.

Sol: $v^2 = u^2 + 2as$

If $u = 0$, $v^2 \propto s$. ⇒ option (c)

16.

Sol: $u_{balloon,y} = 10 + 5 = 15$ m/s upward

after 2 sec, $v = u - gt$

$= 15 - 10 \times 2 = 5$ m/s downward

17.

Sol: $a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g = 1.4 m/s^2$

18.

Sol: $v = -mx + v_0$

$\frac{dv}{dt} = -m\frac{dx}{dt} \Rightarrow a = -mv = -m(-mx + v_0)$

$a = +m^2x_0 - mv_0$

$a - x$ is st. line with positive slope.

19.

Sol: $\vec{A} = l^2\hat{i}$, $\vec{E} = a\hat{i} + b\hat{j}$

$\phi_E = \vec{E}.\vec{A} = al^2.$

20.

Sol: $E_{av} = L.\frac{\Delta L}{\Delta t} = 50 \times \frac{0.1}{3} = 1.67\ H.$

21.

Sol: KE of rolling body = $\frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$

Also, $v = \sqrt{\frac{2gh}{1 + K^2/R^2}}$

for S.S, $\frac{K^2}{R^2} = \frac{2}{5}$, $v = 10$ m/s.

22.

Sol: $\frac{I_p}{I_s} = \frac{N_s}{N_p} \Rightarrow \frac{4}{I_s} = \frac{280}{140} \Rightarrow I_s = 2A$

23.

Sol: $\frac{d\theta}{dx} = \frac{(125 - 25)^\circ C}{50} = 2\ ^\circ C/cm.$

24.

Sol: 

$\frac{\lambda}{2} = l \Rightarrow \lambda = 2 \times 40 = 80$ cm

25.

Sol: $\because x = a\cos(\omega A + \phi)$

$v = \frac{dx}{dt} = -a\omega\sin(\omega A + \phi)$

acc^n, $A = \frac{dv}{dt} = -a\omega^2\cos(\omega A + \phi)$

at $\phi = 0$, acc^n is $\frac{1}{4}$ T out of phase with velocity

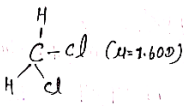
$\Rightarrow \frac{1}{4}T = 90^\circ = 0.5\ s$

$= 5 \times 10\ s \Rightarrow n = 5$

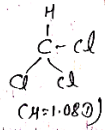
Chemistry

1.

Solv (b) In CH_2Cl_2 there are 2C-H bond and 2C-Cl bonds. Here, the resultant of 2C-H dipole bonds are reduced by the 2C-Cl dipole bond

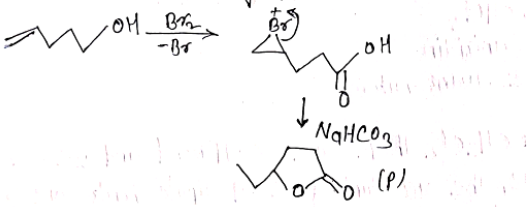


This results in non-zero dipole moment of CH_2Cl_2.



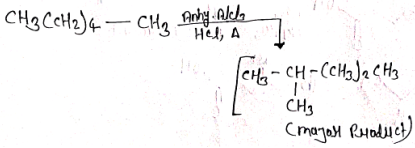
2.

Solv (c) The reaction taking place is as follows



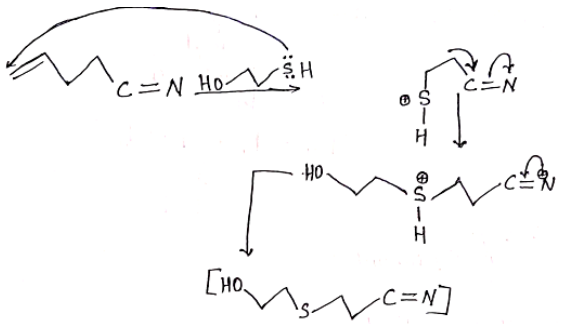
3.

Solv (b) When hexane is heated with anhydrous $AlCl_3$ and HCl gas at 573 K under a pressure of 30-35 atm, it isomerizes to give a branched chain alkane.



4.

Solv

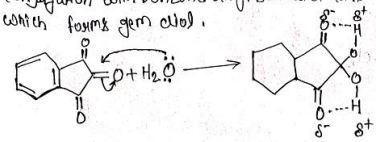


5.

Solv (c) The product [x] formed in the given reaction is given in option (c).

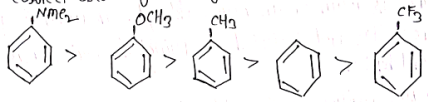
6.

Solv (b) The reactant ninhydrin has three carbonyl groups. Two of them are in conjugation with benzene ring, so water adds to the carbonyl group which forms gem diol.



7.

Solv (b) Correct order of rate of electrophilic aromatic substitution is

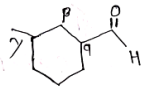


Rate of electrophilic aromatic substitution is directly proportional to the electron density on benzene ring. Electron releasing group (NMe_2 > OCH_3 > CH_3) increase electron density decreases electron density.

8. –

9.

Solv (b)



γ-methylcyclohexane carbaldehyde

cyclohexane is main chain here, aldehyde is fuctional group but since it's not included in principal chain, it is written as carbaldehyde.

Third position to the aldehyde group is called γ-position

10. c

11. c

12. c

13.

Solv (b) $CH_3-\overset{CH_3}{\underset{}{CH}}-CH_2-\underset{CH_2}{\underset{|}{CH}}-COOH$

Proteins on hydrolysis gives α-amino acid because amino acid are the building block of proteins. Among the given options, only in option (b), the protein contains α-amino acids.

14.

Solv (b) Let's take an example of two amino acids
$A = H_2N-\underset{R_1}{CH}-COOH$; $B = H_2N-\underset{R_2}{CH}-COOH$

Tripeptides are three amino acid molecules joined together with the elimination of water molecule, leads to the formation of $\left[H-N-\overset{O}{\overset{\|}{C}}-\right]$ bonds

With two amino acid, four combinations of cyclic tripeptides are possible ie; AAB, ABB, AAA and BBB. Hence, the answer is four.

15.

Solv (a) $K_2 = \frac{1}{t}\ln\frac{100}{4}$

$K_1 = \frac{1}{t}\ln\frac{100}{50}$

$\frac{K_2}{K_1} = \frac{\ln 25}{\ln 2} = 4.65$

16.

Solv (a) $2N_2O_5(CCl_4) \longrightarrow 2N_2O_4(CCl_4) + O_2(g)$

$t=0$	a mol	0	
$t=t$	$a-x$	$\frac{x}{2}$ mol	$\frac{x}{2}$ mol
$t=\infty$	0		

$a \Rightarrow 100$
$x \Rightarrow 90$

$K = \frac{1}{t}\ln\frac{a}{a-x} = \frac{2.303}{500}$

17. d
18. b
19. a
20. c
21.

$NH_3(g) \rightleftharpoons N_2(g) + 3H_2(g)$

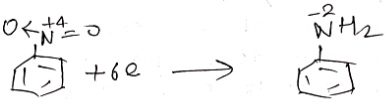
Rate $= k[NH_3]^0$

Rate $= 2$

Rate $= \frac{1}{3} \times \frac{\Delta[H_2]}{\Delta t} \Rightarrow \frac{\Delta[H_2]}{\Delta t} = 6$

22.

Solv



6 F Ans

23.

Soln $P = \frac{2}{5}\times 5 + \frac{3}{5}\times 10 = 8$ torr.

24.

Soln (4 9)

E.C of Fe : $[Ar]\ 4s^2\ 3d^6$

$n = 4$

Spin only mag moment $= \sqrt{n(n+2)}$ B.M

$= \sqrt{4(4+2)} = \sqrt{24}$

$= 4.9$

$= 49\times10^{-1}$

25.

Soln (4) An oxide which combine with water to form an acid generally. the oxides of non metals are acidic in nature hence N_2O_3, NO_2, Cl_2O_7 and SO_2 are acidic except N_2O, NO, CO which are neutral oxides.

Math

1.

① LHL,

$\lim_{x\to0^-}\frac{1-\cos3x}{x^2} = \lim_{x\to0^-}\frac{1-\cos3x}{(3x)^2}\times9 = \frac{9}{2}$

RHL: $\lim_{x\to0^+}\frac{B\sqrt{1-\cos2x}}{x} = \frac{B\sqrt{2}\sin x}{x} = B\sqrt{2}$

f is continuous so

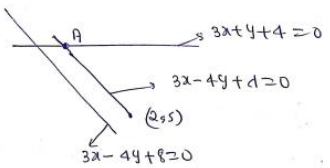
$\frac{9}{2} = \alpha = B\sqrt{2}$

$\alpha = \frac{9}{2}$, $B = \frac{9}{2\sqrt{2}}$

$\alpha^2 = \frac{81}{4}$ $B^2 = \frac{81}{8} \Rightarrow 4\alpha^2 + 8B^2 = 162$

2.

②



$3x-4y+1=0$ passes through $(2,5)$ so

$6-20+1=0 \Rightarrow \boxed{1=14}$

Point of intersection of $3x+y+4=0$ and $3x-4y+14=0$

$3x+y+4=0$

$3x-4y+14=0$

$5y-10=0$

$y=2 \Rightarrow 3x+2+4=0$

$x=-2$

Point $A(-2,2)$

Distance $= \sqrt{(2+2)^2+(5-2)^2} = \sqrt{16+9} = 5$ Ans

3.

③ Given

$$(x^2-1)\frac{dy}{dx}+2xy=\frac{1}{x^2-1}$$

$$\frac{dy}{dx}+\left(\frac{2x}{x^2-1}\right)y=\frac{1}{(x^2-1)^2}$$

$$\frac{dy}{dx}+P(x)y=Q(x) \longrightarrow \text{LDE}$$

$$P(x)=\frac{2x}{x^2-1} \qquad Q(x)=\frac{1}{(x^2-1)^2}$$

$$IF=e^{\int\frac{2x}{x^2-1}dx} \Rightarrow \text{let } x^2-1=u,\ 2x\,dx=du$$

$$\Rightarrow e^{\int\frac{du}{u}}=e^{\ln u}=e^{\ln(x^2-1)}=x^2-1$$

$$IF=x^2-1$$

so $$y(x^2-1)=\int\frac{1}{x^2-1}dx+C$$

$$y(x^2-1)=\frac{1}{2}\int\frac{2}{(x-1)(x+1)}dx+C=\frac{1}{2}\int\frac{(x+1)-(x-1)}{(x-1)(x+1)}dx+C$$

$$\Rightarrow\frac{1}{2}\left(\frac{1}{x-1}-\frac{1}{x+1}\right)dx+C$$

$$\Rightarrow\frac{1}{2}\left(\ln|x-1|-\ln|x+1|\right)+C$$

$$\boxed{y(x^2-1)\Rightarrow\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+C}$$

4.

④ $|x^2+4x+9+2x-3|=|x^2+4x+9|+|2x-3|$

$|a+b|=|a|+|b|$

This equation is valid only when a and b both have same sign.

(i) $x^2+4x+9=(x+2)^2+5>0$

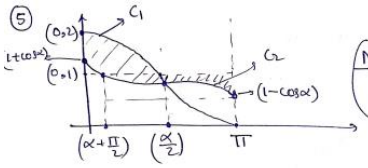
so $(2x-3)$ should be also positive

$$2x-3\geq 0$$

$$\boxed{x\geq\frac{3}{2}}$$

5.

⑤



Note:- Just a Rough sketch Graph may vary

POI of C_1 & C_2: $1+\cos x=1+\cos(x-\alpha)$

$\cos x=\cos(x-\alpha)$

$x=2n\pi+(x-\alpha)$

$x=\pm(x-\alpha)\Rightarrow$ ~~$x=\alpha$~~ $\boxed{x=\frac{\alpha}{2}}$

POI of $y=1$ & C_2: $1+\cos(x-\alpha)=1$

$\cos(x-\alpha)=0$

$\boxed{x=\alpha+\frac{\pi}{2}}$

(i) area between C_1, C_2 & $x=0$

$$A_1=\int_0^{\alpha/2}\left[1+\cos x-1-\cos(x-\alpha)\right]dx=\int_0^{\alpha/2}\left(\cos x-\cos(x-\alpha)\right)dx$$

$$\Rightarrow\left[\sin x-\sin(x-\alpha)\right]_0^{\alpha/2}=2\sin\frac{\alpha}{2}-\sin\alpha$$

(ii) area between C_2, $y=1$, $x=\pi$

$$A_2=\int_{\alpha+\frac{\pi}{2}}^{\pi}\left(1+\cos(x-\alpha)-1\right)dx=\int_{\alpha+\pi/2}^{\pi}\cos(x-\alpha)\,dx=\sin(x-\alpha)\Big]_{\alpha+\frac{\pi}{2}}^{\pi}$$

$A_2=\sin\alpha-1\Rightarrow$ area is always positive so

$$\boxed{A_2=1-\sin\alpha}$$

given $A_1=A_2$

$$2\sin\frac{\alpha}{2}-\sin\alpha=1-\sin\alpha$$

$$\sin\left(\frac{\alpha}{2}\right)=\frac{1}{2}=\sin$$

$$\frac{\alpha}{2}=\frac{\pi}{6}\Rightarrow\boxed{\alpha=\frac{\pi}{3}}$$

6.

⑥

$$Q=\begin{bmatrix}2^2a_{11} & 2^3a_{12} & 2^4a_{13}\\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23}\\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33}\end{bmatrix}$$

$$|Q|=\begin{vmatrix}2^2a_{11} & 2^3a_{12} & 2^4a_{13}\\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23}\\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33}\end{vmatrix}$$

Take common, 2^2 from R_1, 2^3 from R_2, 2^4 from R_3

$$|Q|=2^9\begin{vmatrix}a_{11} & 2a_{12} & 2^2a_{13}\\ a_{21} & 2a_{22} & 2^2a_{23}\\ a_{31} & 2a_{32} & 2^2a_{33}\end{vmatrix}$$

Take common 2 from C_2, 2^2 from C_3

$\Rightarrow 2^{12}|P|\Rightarrow$ $\boxed{2^{13}}$ ans

7.

⑦

$$\int\frac{\cos x+\sqrt{3}}{1+4\sin\left(x+\frac{\pi}{3}\right)+4\sin^2\left(x+\frac{\pi}{3}\right)}dx$$

$$\int\frac{\cos x+\sqrt{3}}{\left[1+2\sin\left(x+\frac{\pi}{3}\right)\right]^2}dx$$

$$\int\frac{\cos x+\sqrt{3}}{\left(1+2\sin x\cos\frac{\pi}{3}+2\cos x\sin\frac{\pi}{3}\right)^2}dx$$

$$\int\frac{\cos x+\sqrt{3}}{\left(1+\sin x+\sqrt{3}\cos x\right)^2}dx$$

Divide by $\sin^2 x$

$$\int\frac{\cot x\operatorname{cosec}x+\sqrt{3}\operatorname{cosec}^2x}{\left(\operatorname{cosec}x+1+\sqrt{3}\cot x\right)^2}dx$$

$\operatorname{cosec}x+1+\sqrt{3}\cot x=u$

$(-\operatorname{cosec}x\cot x-\sqrt{3}\operatorname{cosec}^2x)dx=du$

$$\int\frac{-du}{u^2}\Rightarrow\frac{1}{u}+C\Rightarrow\frac{1}{\operatorname{cosec}x+1+\sqrt{3}\cot x}+C$$

$$\Rightarrow\frac{\sin x}{1+\sin x+\sqrt{3}\cos x}+C\Rightarrow\boxed{\frac{\sin x}{1+2\sin\left(x+\frac{\pi}{3}\right)}+C}$$

ans

8.

(8) $P(x)=\left(\sqrt{x^5-1}+x\right)^7-\left(\sqrt{x^5-1}-x\right)^7$

$\Rightarrow (y+x)^7-(y-x)^7$

$\Rightarrow {}^7C_0y^7+{}^7C_1y^6x+{}^7C_2y^5x^2+\cdots+{}^7C_7x^7$

$-{}^7C_0y^7+{}^7C_1y^6x-{}^7C_2y^5x^2+\cdots-{}^7C_7x^7$

$P(x)=2\left({}^7C_1y^6x+{}^7C_3y^4x^3+\cdots+{}^7C_7x^7\right)$

$\Rightarrow 2\left({}^7C_1\left(\sqrt{x^5-1}\right)^6x+{}^7C_3\left(\sqrt{x^5-1}\right)^4x^3+\cdots+{}^7C_7x^7\right)$

$\Rightarrow 2\left({}^7C_1(x^5-1)^3x+{}^7C_3(x^5-1)^2x^3+\cdots+{}^7C_7x^7\right)$

Highest Power of $x \Rightarrow$ (16)

9.

(9) Let the numbers selected be x_1, x_2, x_3

$2x_2=x_1+x_3$

$\Rightarrow x_1+x_3 \Rightarrow$ even

(i) x_1 and x_3 both are even $= {}^{12}C_2$ ways

(ii) x_1 and x_3 both are odd $\Rightarrow {}^{12}C_2$ ways.

Total $= {}^{12}C_2+{}^{12}C_2=132$

10.

(10) Let assume that A wins after n deuces, $n\in(0,\infty)$

Probability of a deuce $=\frac{2}{3}\cdot\frac{2}{3}+\frac{1}{3}\cdot\frac{1}{3}=\frac{5}{9}$

(A wins his serve then B wins his serve or A loses his serve then B also loses his serve) Now probability of A winning the game

$$\sum_{n=0}^{\infty}\left(\frac{5}{9}\right)^n\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)=\left(\frac{1}{1-5/9}\right)\frac{2}{9}=\frac{1}{2}$$ Hence

11.

(11) $\vec{\alpha}\cdot\vec{\beta}=ab+bc+ca$

$|\alpha||\beta|\cos\theta=ab+bc+ca$

$\cos\theta=\frac{ab+bc+ac}{|\alpha||\beta|}$

$|\alpha|=|\beta|=\sqrt{a^2+b^2+c^2}$

$\cos\theta=\frac{ab+bc+ac}{a^2+b^2+c^2}$.

we know

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)\geq 0$

$\frac{ab+bc+ca}{a^2+b^2+c^2}\geq -\frac{1}{2}$

$\cos\theta\geq -\frac{1}{2} \Rightarrow \theta\in\left[0,\frac{2\pi}{3}\right]$

12.

(12) $\sum_{x=1}^{1029}[\log_2^x]=\sum_{x=1}^{2^{10}}[\log_2^x]$

$\Rightarrow \sum_{x=1}^{2^2-1}[\log_2^x]+\sum_{x=2^2}^{2^3-1}[\log_2^x]+\cdots+\sum_{x=2^9}^{2^{10}-1}[\log_2^x]+\log_2^{2^{10}}$

$\Rightarrow 2\cdot1+2^2\cdot2+2^3\cdot3+2^9\cdot9+\cdots+2^9\cdot9+10=8209$ Hence

$\Rightarrow$

13.

(13) $\bar{x}=8$, $\sigma^2=16$

Let two observations, a & b

$\bar{x}=\frac{\sum x_i}{7}\Rightarrow \sum x_i=56$

$\Rightarrow a+b+42=56$

$a+b=14$

$\sigma^2=\frac{\sum x_i^2}{n}-(\bar{x})^2=16$

$\frac{\sum x_i^2}{7}-64=16$

$\frac{\sum x_i^2}{7}=80$

$\sum x_i^2=560$

$2^2+4^2+10^2+12^2+14^2+a^2+b^2=560$

$a^2+b^2=100$

$(a+b)^2=a^2+b^2+2ab$

$14^2=100+2ab$

$\frac{196-100}{2}=ab$ $ab=48$

14.

(14) $\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=a^2-b^2$ —(i)

$\frac{ax\sin\theta}{\cos^2\theta}-\frac{by\cos\theta}{\sin^2\theta}=0$ —(ii)

Using Eq^n (ii)

$\frac{ax\sin\theta}{\cos^2\theta}=\frac{by\cos\theta}{\sin^2\theta}$

$\frac{ax}{by}=\frac{\cos^3\theta}{\sin^3\theta}$

$\tan^3\theta=\frac{by}{ax}$

$\tan\theta=\left(\frac{by}{ax}\right)^{1/3}$

Let $ax=m$ $by=n$

$\tan\theta=\left(\frac{n}{m}\right)^{1/3}$

or $n=m\tan^3\theta$ —(iii)

Using Eq^n (i)

$\frac{m}{\cos\theta}+\frac{n}{\sin\theta}=a^2-b^2$

$\frac{m}{\cos\theta}+\frac{m\tan^3\theta}{\sin\theta}=a^2-b^2$

$m\left(\frac{1}{\cos\theta}+\frac{\tan^3\theta}{\sin\theta}\right)=a^2-b^2$

$m\left(\frac{\cos^2\theta+\sin^2\theta}{\cos^3\theta}\right)=a^2-b^2$

$m=(a^2-b^2)\cos^3\theta$

$n=(a^2-b^2)\sin^3\theta$

we have to find

$m^{2/3}+n^{2/3}=(a^2-b^2)^{2/3}\cos^2\theta+(a^2-b^2)^{2/3}\sin^2\theta$

$\Rightarrow (a^2-b^2)^{2/3}$ Hence

15.

(15) Let $z = x+iy$

$$\text{Re}\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right) = \text{Re}\left(\frac{x+iy-(x-iy)+x^2+y^2}{2-3(x+iy)+5(x-iy)}\right)$$

$$\Rightarrow \text{Re}\left(\frac{x^2+y^2+i(2y)}{2+2x-8iy}\right)$$

$$\Rightarrow \text{Re}\left(\frac{[x^2+y^2+i(2y)][2(1+x)+8iy]}{(2+2x-8iy)(2+2x+8iy)}\right)$$

$$\Rightarrow \frac{2(x^2+y^2)(x+y)-16y^2}{4(1+x)^2+(8y)^2}$$

Given $\text{Re}(z) = x = 3$

$$\Rightarrow \frac{8(9+y^2)-16y^2}{64+64y^2} = \frac{1}{8}\frac{(9-y^2)}{1+y^2} \quad 2\alpha$$

$$8\beta+8\beta y^2 = 9-y^2$$

$$y^2(8\beta+1)+8\beta-9=0$$

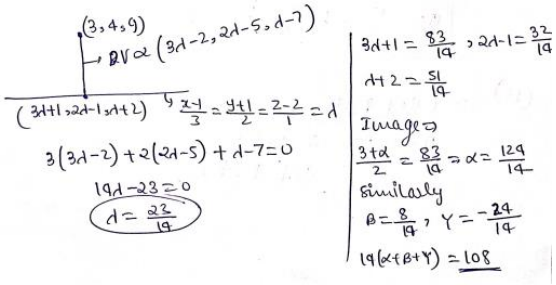
now, $D \geq 0$

$$0^2-4(8\beta+1)(8\beta-9) \geq 0$$

$$\beta \in \left(-\frac{1}{8}, \frac{9}{8}\right] \qquad \text{Ans} \Rightarrow 29(\beta-\alpha) = 30$$

16.

(16)



$$3(3\lambda-2)+2(2\lambda-5)+\lambda-7=0$$

$$14\lambda-23=0$$

$$\lambda = \frac{23}{14}$$

$3\lambda+1 = \frac{83}{14}$, $2\lambda-1 = \frac{32}{14}$

$\lambda+2 = \frac{51}{14}$

Image ⇒

$$\frac{3+\alpha}{2} = \frac{83}{14} \Rightarrow \alpha = \frac{129}{14}$$

similarly

$\beta = \frac{8}{14}$, $\gamma = -\frac{24}{14}$

$14(\alpha+\beta+\gamma) = 108$

17.

(17) $C_1(0,0)$ $r_1 = 2$ $C_2 = (2\lambda, 0)$ $r_2 = \sqrt{4\lambda^2-9}$

$\lambda \in \left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

To intersect at two points.

$|r_1-r_2| < C_1C_2 < r_1+r_2$

(i) $C_1C_2 < r_1+r_2$

$|2\lambda| < 2+\sqrt{4\lambda^2-9}$

$4\lambda^2 < 4+4\lambda^2-9+4\sqrt{4\lambda^2-9}$

$4\sqrt{4\lambda^2-9} > 5$

$16(4\lambda^2-9) > 25$

$64\lambda^2-144 > 25$

$\lambda^2 - \frac{169}{64} > 0$

$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$

(ii) $C_1C_2 > |r_1-r_2|$

$|2\lambda| > |2-\sqrt{4\lambda^2-9}|$

$4\lambda^2 > 4+4\lambda^2-9-4\sqrt{4\lambda^2-9}$

$4\sqrt{4\lambda^2-9} > -5$

$\forall\ x \in \left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

Taking intersection.

$$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$$

$a = -\frac{13}{8}$, $b = \frac{13}{8}$ → solve ahead

18.

(18) $x^2+ax+4 \geq 0$

$D \leq 0 \Rightarrow a^2-16 \leq 0$

$a \in [-4, 4]$.

19.

(19) $\left[x-\frac{1}{2}\right]\left[x+\frac{1}{2}\right] \Rightarrow$ Prime

$\left(x+\frac{1}{2}\right)-\left(x-\frac{1}{2}\right) = 1$ Difference between both number

Possible combination:

$\left[x-\frac{1}{2}\right] = 1$ and $\left[x+\frac{1}{2}\right] = 2$

$1 \leq x-\frac{1}{2} < 2$ and $2 \leq x+\frac{1}{2} < 3$

$\frac{3}{2} \leq x < \frac{5}{2}$ and $\frac{3}{2} \leq x < \frac{5}{2}$

(OR)

$\left[x-\frac{1}{2}\right] = -2$ and $\left[x+\frac{1}{2}\right] = -1$

$-2 \leq x-\frac{1}{2} < -1$ and $-1 \leq x+\frac{1}{2} < 0$

$-\frac{3}{2} \leq x < -\frac{1}{2}$ and $-\frac{3}{2} \leq x < -\frac{1}{2}$

$x_1 = -\frac{3}{2}$ $x_2 = -\frac{1}{2}$ $x_3 = \frac{3}{2}$ $x_4 = \frac{5}{2}$

20.

(20) $\text{tr}(AB) \Rightarrow$ sum of diagonal elements of AB

$\text{tr}(AB) = 4\sec^2\theta\cot^2\theta+1+9+\tan^2\theta\,\text{cosec}^2\theta+1+1+4$

$\text{tr}(AB) = 7+4\,\text{cosec}^2\theta+9\sec^2\theta$

$\Rightarrow 7+4(1+\cot^2\theta)+9(1+\tan^2\theta)$

$\Rightarrow 20+\underbrace{4\cot^2\theta+9\tan^2\theta}_{\geq 12}$

use AM ≥ GM

$\frac{4\cot^2\theta+9\tan^2\theta}{2} \geq (4\cot^2\theta \cdot 9\tan^2\theta)^{1/2}$

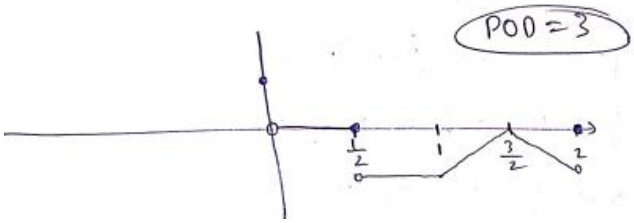
$\Rightarrow \text{tr}(AB) \geq 32$

21.

$$f(x)=\begin{cases}[\cos \pi x) & x\in[0,1] \\ |2x-3|[x-2] & x\in(1,2]\end{cases}$$

$$f(x)=\begin{cases}1 & x=0 \\ 0 & x\in(0,\frac{1}{2}] \\ -1 & x\in(\frac{1}{2},1] \\ 2x-3 & x\in(1,\frac{3}{2}) \\ 0 & x=\frac{3}{2} \\ -(2x-3) & x\in(\frac{3}{2},2) \\ 0 & x=2\end{cases}$$

POD = 3



22.

$$\left(x+\frac{1}{x}\right)^{11}=\sum_{r=0}^{11}{}^{11}C_r (x)^r \left(\frac{1}{x}\right)^{11-r}$$

$$\Rightarrow \sum_{r=0}^{11}{}^{11}C_r (x^r)(x^{r-11}) \Rightarrow \sum_{r=0}^{11}{}^{11}C_r\, x^{2r-11}$$

$2r-11 \longrightarrow$ is an odd number

so there is no even power of x

dus = 0

23.

$$f(x)=\int_0^x \frac{dt}{[f(t)]^2}$$

$$f'(x)=\frac{1}{[f(x)]^2} \Rightarrow [f(x)]^2 f'(x)=1$$

Integrate both side

$$\int [f(x)]^2 f'(x)\,dx=\int dx$$

$f(x)=u$

$f'(x)dx=du$

$\int u^2 du = x+c$

$\frac{u^3}{3}=x+c$

$[f(x)]^3=3x+c$

$$f(x)=\int_0^x \frac{dt}{[f(t)]^2}$$

$$f(2)=\int_0^2 \frac{dt}{[f(t)]^2}=(6)^{1/3}$$

$f(2)=6^{1/3}$

Put $x=2$

$(f(2))^3=6+c$

$6=6+c$

$c=0$

$[f(x)]^3=3x$

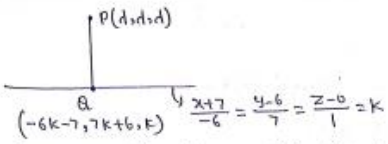
so $x=9$

$[f(9)]^3=27$

$f(9)=3$ dus

24.

④



$\frac{x+7}{-6}=\frac{y-6}{7}=\frac{z-0}{1}=k$

~~PQ = 6~~ $\vec{PQ}=(-6k-7-d)\hat{i}+(7k+6-d)\hat{j}+(k-d)\hat{k}$

$\vec{PQ}$ is perpendicular to lines

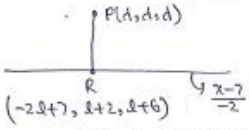
$(-6k-7-d)(-6)+(7k+6-d)7+k-d=0$

$36k+42+6d+49k+42-7d+k-d=0$

$86k+84-2d=0$

$43k+42-d=0$

$k=\frac{d-42}{43}$



$\frac{x-7}{-2}=\frac{y-2}{1}=\frac{z-6}{1}=l$

$\vec{PR}=(-2l+7-d)\hat{i}+(l+2-d)\hat{j}+(l+6-d)\hat{k}$

$\vec{PR}$ is perpendicular to lines

$(-2l+7-d)(-2)+(l+2-d)+(l+6-d)=0$

$4l-14+2d+l+2-d+l+6-d=0$

$6l-6=0$

$l=1$

$\vec{PR}=(5-d)\hat{i}+(3-d)\hat{j}+(7-d)\hat{k}$

~~$\frac{d+1}{43}$~~ $\vec{PQ}=-\left(\frac{d+1}{43}\right)(49\hat{i}+36\hat{j}+42\hat{k})$

$\vec{PQ}\perp\vec{PR}$

$(5-d)49+(3-d)36+(7-d)42=0$

$245-49d+108-36d+294-42d=0$

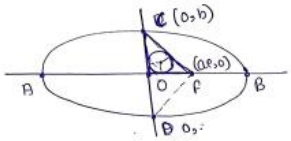
$647=127d \Rightarrow d=\frac{647}{127}$

$d+1=0$

$d=-1$

25.

(10)



$ae=6$

radius of circle = 1

so center (1,1)

$b^2=a^2(1-e^2)$

$b^2=a^2-(ae)^2$

$a=\sqrt{b^2+(ae)^2}$

$\Rightarrow \sqrt{36+6.25} \Rightarrow \sqrt{42.25}$

$a=6.5$

Eq^n of CF

$\Rightarrow \frac{x}{ae}+\frac{y}{b}=1 \Rightarrow \frac{x}{ae}+\frac{y}{b}-1=0$

~~Position~~ Distance of this from (1,1) is 1

$$1=\frac{\left|\frac{1}{6}+\frac{1}{b}-1\right|}{\sqrt{\frac{1}{36}+\frac{1}{b^2}}}$$

$AB=13$

$CD=5$

$\frac{AB\cdot CD}{13}=5$

dus

$\frac{1}{36}+\frac{1}{b^2}=\frac{1}{36}+\frac{1}{b^2}+1+\frac{1}{3b}-\frac{2}{b}-\frac{1}{3}$

$\frac{6}{3b}-\frac{1}{3b}=\frac{2}{3} \Rightarrow \frac{5}{3b}=\frac{2}{3}$

$b=2.5$